1 Preliminaries

In $\mathbb{C}$, every open domain admits an analytic function that cannot be extended across its boundary. In several variables, Hartogs’ domains exhibit a break with this behavior, however domains of holomorphy capture such commonality with one variable.

![Diagram](circle.png)

In separable Banach spaces $X$ with the BAP, the global property of being a domain of holomorphy is described in local analytic/geometric terms by pseudoconvexity [Levi, 1910-1911]: Given a defining function of the boundary, the Levi condition holds at each boundary point $a$:

$$D'D''r(a)(b, b) \geq 0 \text{ for all } b \in \mathbb{C}^n \text{ such that } D'r(a)b = 0. \quad (1)$$

If there is strict inequality for $b \neq 0$, the domain is called strictly pseudoconvex. In finite dimension, the latter are locally biholomorphic to strongly convex sets.

In the 1940s, Oka and Lelong characterized and extended pseudoconvexity as convexity w. r. t. plurisubharmonic functions: upper semicontinuous functions $f : U \to [-\infty, \infty)$, s. t. for each $a \in U$ and $b \in X$ with $a + D \cdot b \subset U$,

$$f(a) \leq \frac{1}{2\pi} \int_0^{2\pi} f(a + e^{it}b) d\theta.$$ 

If $f \in C^2(U, \mathbb{R})$, it is plurisubharmonic iff for all $a \in U$ and $b \in X$, $D'D''f(a)(b, b) \geq 0$; and it is strictly plurisubharmonic if there is strict inequality for $b \neq 0$. In fact, a domain $U$ in $\mathbb{C}^n$ with $C^2$ boundary is strictly pseudoconvex iff there is a strictly p. s. h. function defining the boundary.

While an open domain is convex iff $-\log d_U$ is a convex function, in turn $U$ is pseudoconvex iff $-\log d_U$ is plurisubharmonic. In finite dimension, pseudoconvexity is equivalent to the existence of a plurisubharmonic exhaustion function of the domain.

2 Novelties on strong pseudoconvexity

**Theorem (O-C, ’14).** If $X$ is a 2-uniformly PL-convex Banach space then $L_X$ is uniformly pseudoconvex.

For $p \in [1, 2]$, $B_{L(X,\Omega,\mu)}$ is 2-uniformly PL-convex [Davis, Garling, Tomczak-Jaegermann, ’84]. Meanwhile, for $2 < p \leq \infty$ and $n \geq 2$, the ball of $\ell_p^n$ lacks strong pseudoconvexity, and so do the balls of $L_p$ and $\ell_p$ for $p > 2$. The same prop. fails for $B_{C(K)}$, for $K$ compact and Hausdorff with $|K| \geq 2$ (it contains the polydisk $B_{\ell_2^n}$).

![Diagram](circle.png)