Tables for Strong Pseudoconvexity in Banach Spaces

Table 1: Plurisubharmonicity notions

Concept	Conditions	Equation	Examples
Plurisubharmonic $f : U \to [-\infty, \infty)$	upper semicont.	$f(a) \leq \frac{1}{2\pi}\int_0^{2\pi} f(a+e^{i\theta}b)d\theta$ $a \in U$, b arbitrary	$\mathrm{Re}(F)$, $\mathrm{Im}(F)$, $\\|F\\|_p^m$, $\log(\\|F\\|_p)$ for F holomorphic and $1 \leq p \leq \infty$
C^2 plurisubharmonic f on U	C^2 smooth	$\sum_{j,k=1}^n \frac{\partial^2 f}{\partial z_j \partial \bar{z}_k}(a) b_j \bar{b}_k \geq 0$ $a \in U$, b arbitrary	$\mathrm{Re}(F)$, $\mathrm{Im}(F)$, $\\|F\\|_p^m$, $\log(\\|F\\|_p)$ for F holomorphic and $1 < p < \infty$
Strictly p.s.h. on av. $f : U \to [-\infty, \infty)$	f upper semicont.	$\varphi(a)\\|b\\|^2 + f(a) \leq \frac{1}{2\pi}\int_0^{2\pi} f(a+e^{i\theta}b)d\theta$ $\varphi > 0 \in C^\infty(U)$, $a \in U$, $\\|b\\|$ small	$\\|\cdot\\|_p$ for $1 \leq p \leq 2$, $\sum_{k=1}^\infty \lvert z_k\rvert^2/k^3$: $\sum_{k=1}^\infty \lvert z_k\rvert^2/k^2 < \infty$
C^2 strictly p.s.h. f on U	C^2 smooth	$\sum_{j,k=1}^n \frac{\partial^2 f}{\partial z_j \partial \bar{z}_k}(a) b_j \bar{b}_k \geq \varphi(a)\\|b\\|^2$ $\varphi > 0 \in C^\infty(U)$, $a \in U$, b any	$\\|\cdot\\|_p$ for $1 < p \leq 2$ outside $\{0\}$, $\sum_{k=1}^\infty \lvert z_k\rvert^2/k^3$: $\sum_{k=1}^\infty \lvert z_k\rvert^2/k^2 < \infty$
Strongly p.s.h. on av.	upper semicont.	$L\\|b\\|^2 + f(a) \leq \frac{1}{2\pi}\int_0^{2\pi} f(a+e^{i\theta}b)d\theta$	$\\|\cdot\\|_p$ for $1 \leq p \leq 2$
C^2 strongly p.s.h.	C^2 smooth	$\sum_{j,k=1}^n \frac{\partial^2 f}{\partial z_j \partial \bar{z}_k}(a) b_j \bar{b}_k \geq L\\|b\\|^2$	$\\|\cdot\\|_p$ for $1 < p \leq 2$ outside $\{0\}$

Table 2: Pseudoconvexity in $\mathbb{C}^n$

Concept	Conditions	Equation	Examples
Ψ-convex domain	inc. union of C^2 Ψ-convex domains	$\sum_{j,k=1}^n \frac{\partial^2 r_n}{\partial z_j \partial \bar{z}_k}(p) \zeta_j \bar{\zeta}_k \geq 0$ $\zeta \in T_p^{\mathbb{C}}(b\Omega_n), p \in b\Omega_n$	convex domains & domains of convergence
C^2 Ψ-convex domain	C^2 smooth boundary def. by r	$\sum_{j,k=1}^n \frac{\partial^2 r}{\partial z_j \partial \bar{z}_k}(p) \zeta_j \bar{\zeta}_k \geq 0$ $\zeta \in T_p^{\mathbb{C}}(b\Omega)$, $p \in b\Omega$	$B_{\ell_p^n}$, $1 < p < \infty$; $\{z \in \mathbb{C}^2 : \lvert z_1 z_2\rvert < 1\}$; $\{z \in \mathbb{C} : \mathrm{Im} z \leq (\mathrm{Re} z)^2\}$
C^2 strictly Ψ-convex	C^2 smooth boundary def. by r	$\sum_{j,k=1}^n \frac{\partial^2 r}{\partial z_j \partial \bar{z}_k}(p) \zeta_j \bar{\zeta}_k > 0$ $\zeta \in T_p^{\mathbb{C}}(b\Omega)$, $p \in b\Omega$	$B_{\ell_p^n}$, $1 < p \leq 2$
C^2 strongly Ψ-convex	C^2 smooth boundary def. by r	$\sum_{j,k=1}^n \frac{\partial^2 r}{\partial z_j \partial \bar{z}_k}(p) \zeta_j \bar{\zeta}_k > L\\|\zeta\\|^2$ & $\lvert\nabla r(p)\rvert \geq M$; $p \in U \supset b\Omega$	$B_{\ell_p^n}$, $1 < p \leq 2$
Strongly Ψ-convex	inc. union of C^2 str. Ψ-convex w. same bounds$^+$	$\sum_{j,k=1}^n \frac{\partial^2 r_n}{\partial z_j \partial \bar{z}_k}(p) \zeta_j \bar{\zeta}_k > L\\|\zeta\\|^2$ & $\lvert\nabla r_n(p)\rvert \geq M$; $p \in U \supset b\Omega$	$B_{\ell_p^n}$, $1 \leq p \leq 2$

Table 3: Strict Pseudoconvexity in Banach spaces

Concept	Conditions	Equation	Examples
Strictly Ψ-convex $\Omega \subset X$ Banach sp.	$\Omega \cap F$ strongly Ψ-convex, if $\dim F < \infty$	$r_F : U \supset b\Omega \cap F \to [-\infty, \infty)$ str. p.s.h. on av. def. $b\Omega$ & $r_F = \inf_{n\in\mathbb{N}}(r_F)_n$, $\lvert\nabla(r_F)_n\rvert \geq M$	B_{ℓ_p}, $1 \leq p \leq 2$
Strongly Ψ-convex $\Omega \subset X$ Banach sp.	$b\Omega$ is def. around $p \in b\Omega$ by $r_p : U_p \ni p \to [-\infty, \infty)$	$L\lVert w\rVert^2 + r_p(z) \leq \frac{1}{2\pi}\int_0^{2\pi} r_p(z + e^{i\theta}w)d\theta$ $z \in U_p$, $w \in X$	B_{ℓ_p}, $1 \leq p \leq 2$
Uniformly Ψ-convex $\Omega \subset X$ Banach sp.	$b\Omega$ is def. by $r : U \supset b\Omega \to [-\infty, \infty)$	$L\lVert w\rVert^2 + r(z) \leq \frac{1}{2\pi}\int_0^{2\pi} r(z + e^{i\theta}w)d\theta$ $z \in U$, $w \in X$	B_{ℓ_p}, $1 \leq p \leq 2$